Section 7.4 Reciprocals of Linear Functions

The reciprocal of a number or expression can be found by writing 1 in the numerator and the number or expression in the denominator.

Examples:

The reciprocal of 4 is $\frac{1}{4}$.

The reciprocal of x+2 is $\frac{1}{x+2}$.   (This is the reciprocal of a Linear Function.)

The reciprocal of $x^2 + 3x + 2$ is $\frac{1}{x^2+3x+2}$. (This is the reciprocal of a Quadratic Function.)

Graph y = x and y = $\frac{1}{x}$ on the same grid. Use a Table of Value for each Function.

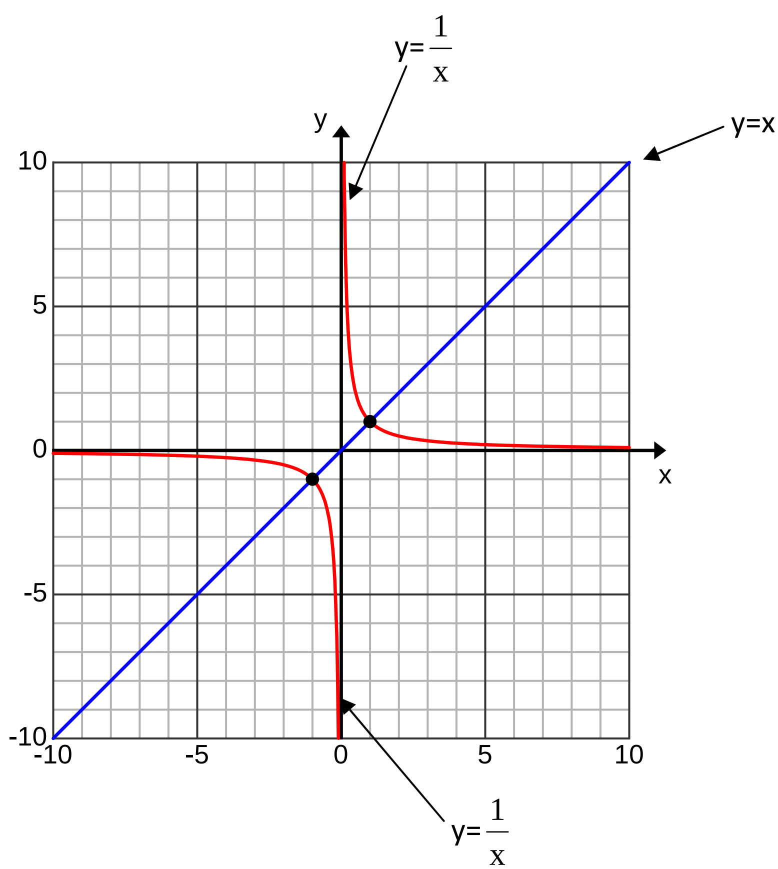
Note: ∞ is the symbol for undefined.

y = x

x	y
4	4
2	2
1	1
½	½
¼	¼
0	0
-¼	-¼
-½	-½
-1	-1
-2	-2
-4	-4

y = $\frac{1}{x}$

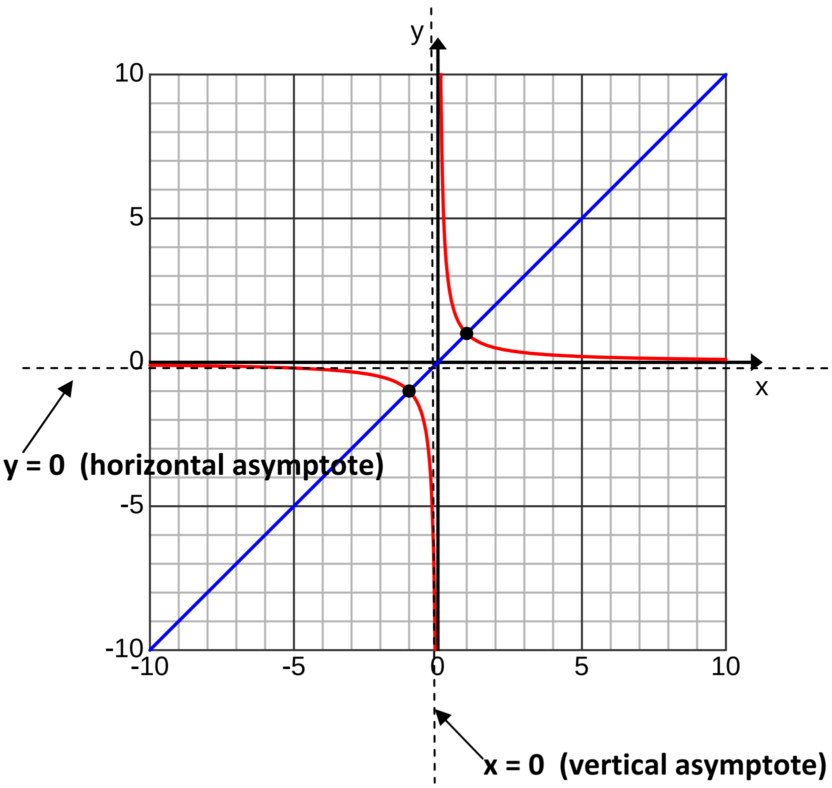
x	y
4	¼
2	½
1	1
½	2
¼	4
0	∞
-¼	-4
-½	-2
-1	-1
-2	-½
-4	-¼



Notice that **when y = 1 or y = -1**, then the points are on both functions.
We call these the "**invariant**" points for both graphs.
The two functions will intersect at these invariant points.

Asymptotes:
An asymptote is a straight line that is approached, but never reached by a curve. They are identified by a dashed line on the graph.

In the graph below, there is a vertical and a horizontal asymptote.



Vertical Asymptotes are drawn where the x values are Not Permissible (NPV's of x).

In this example y = $\frac{1}{x}$, x cannot be 0,

so **x = 0 is the vertical asymptote**.

Horizontal Asymptotes are drawn where the y values are Not Permissible.

For reciprocal functions y=$\frac{1}{??}$, y can never be 0, so **y = 0 is the horizontal asymptote**.

Example1: Graph f(x) = 2x – 3 and then graph its reciprocal.
Label the asymptotes, the invariant points and the intercepts of the reciprocal

Step1: Graph the function y = 2x – 3 (m = 2, b = -3)
Step2: Write out the reciprocal function.

$$y = \frac{1}{2x-3}$$

Step3: Find the horizontal and vertical asymptotes of the reciprocal function.

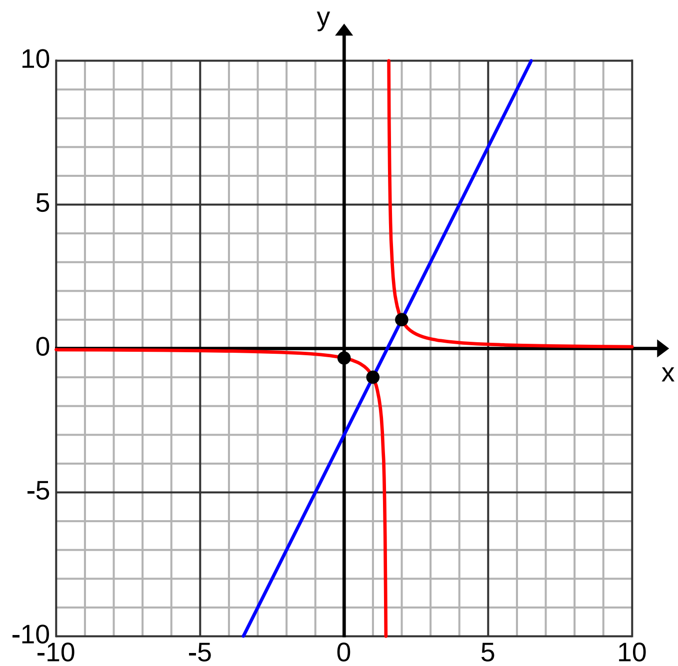
Horiz Asym: is always y = 0 for these types of reciprocal functions y = $\frac{1}{????}$

Vert Asym: Find NPV's for x. (Recall: The denominator can't be zero!!)

$$2x - 3 \neq 0 \quad \text{so} \quad x \neq \frac{3}{2}$$

Draw the dotted lines (asymptotes) for **y = 0** and for **x =** $\frac{3}{2}$

Step4: Find the invariant points.



Recall: Invariant points are on both graphs.
They occur **when y = 1 or when y = -1**.

So solve: 2x – 3 = 1 and 2x – 3 = -1

2x = 4 2x = 2

x = 2 x = 1

The **invariant points are at (2, 1) and at (1, -1).**
Draw these points on the graph.

Step5: Find the intercepts of the reciprocal.

Since the reciprocal never crosses the x-axis,
there are **NO x-intercepts** for the reciprocal.

To find the y-intercept of the reciprocal,
set x = 0 and solve.

y = $\frac{1}{2x-3}$ y = $\frac{1}{2(0)-3} = \frac{1}{-3} = -\frac{1}{3}$

So the y-int of the reciprocal is $-\frac{1}{3}$. Label this point.

Textbook Assignment: Page 403-404 #1ab, 3ab, 4, 5ab, 7bd